[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 5802 **H**

Unique Paper Code : 237301

Name of the Paper : Probability and Statistical Methods – III

Name of the Course : **B.Sc. (Hons.) Statistics**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **five** questions in all selecting **two** from **Section I** and **three** from **Section II.**

SECTION – I

1. (a) Let X and Y be two random variables with variances σ_x^2 and σ_y^2 with correlation coefficient r. If $U = X + KY$ and $V = X + \frac{\sigma_x}{\sigma_y}Y$, find the value of K so that U and V are uncorrelated.

P.T.O.

(b) What is the principle of least squares? Obtain the line of regression of Y on X. (8,7)

2. (a) Define Spearman's rank correlation coefficient. Obtain the value of rank correlation coefficient when each of the deviations is maximum.

(b) If X and Y are independent gamma variates with parameters μ and υ respectively then show that the variables $U = X + Y$ and $V = \dfrac{X-Y}{X+Y}$ are independent variables. (7,8)

3. (a) Prove that in a tri-variate distribution $\sigma_{1.23}^2 = \sigma_1^2 \dfrac{\omega}{\omega_{11}}$.

Hence or otherwise show that $\sigma_1^2 \geq \sigma_{1.2}^2 \geq \sigma_{1.23}^2$.

Where symbols have their usual meaning.

(b) Show that the correlation coefficient between the residuals $X_{1.23}$ and $X_{2.13}$ is equal and opposite of that between $X_{1.3}$ and $X_{2.3}$ (8,7)

SECTION – II

4. (a) State and prove generalised form of Bienayme' Chebychev's inequality. Hence obtain Chebychev's inequality.

(b) If $X_1, X_2, X_3, \ldots, X_n$ are i.i.d random variables with mean μ and variance σ^2 (finite) and $S_n = X_1 + X_2 + \ldots, X_n$, then for $-\infty < a < b < \infty$;

$$\lim_{n\to\infty} P\left[a \leq \frac{S_n - n\mu}{\sigma\sqrt{n}} \leq b\right] \to \Phi(b) - \Phi(a).$$ Where $\Phi(.)$ is the distribution function of standard normal variate. (8,7)

5. (a) Let $X_1, X_2, X_3, \ldots, X_n$ be i.i.d random variables with mean μ and variance σ^2 (finite). If $S_n = X_1 + X_2 + \ldots, X_n$, then examine whether the weak law of large numbers holds for the sequence $\{S_n\}$.

(b) Define convergence in probability, convergence with probability one and convergence in mean square. Prove that convergence in mean square implies convergence in probability.

(c) Let X be a discrete random variable with its characteristic function $(q + pe^{it})^n$. Obtain the probability function of X. (5,5,5)

6. (a) If (X, Y) is distributed as $N(0, 0, 1, 1, \rho)$ with joint p.d.f. $f(x, y)$, then prove that

$$P(X > 0 \cap Y > 0) = \frac{1}{4} + \frac{\sin^{-1}\rho}{2\pi}.$$

(b) Show that if X_1 and X_2 are standard normal variates with correlation coefficient ρ then the correlation coefficient between X_1^2 and X_2^2 is given by ρ^2.

(c) Write short notes on $\sin^{-1}$ transformation and Logarithmic transformation. (5,5,5)

7. (a) Let $\underset{\sim}{X}$ follows $N_p(\mu,\Sigma)$ and further let $\underset{\sim}{X}$ be partitioned as $\underset{\sim}{X} = \begin{pmatrix} \underset{\sim}{X}^{(1)} \\ \underset{\sim}{X}^{(2)} \end{pmatrix}$ in k and (p-k) component sub vectors respectively. Find marginal distribution of $\underset{\sim}{X}^{(1)}$.

(b) If $X_1, X_2, \ldots, X_k$ have multinomial distribution with joint distribution function given by

$$p(x_1, x_2, \ldots, x_k) = \frac{n!}{x_1!x_2!,\ldots, x_k!} p_1^{x_1} p_2^{k_2} \ldots p_k^{x_k}$$

where $0 \le x_i \le n, \ \sum_{i=1}^{k} x_i = n, \ \sum_{i=1}^{k} p_i = 1$.

Then obtain

(i) The marginal distributions of X_i

(ii) Co variance $(X_i, X_j), \ i \ne j$

(iii) $\rho(X_i, X_j)$.

(8,7)